## **4.6** Rules of Exponents



Before Now Why?

You multiplied and divided numerical expressions.

**STEP 1** Copy and complete the table.

You'll multiply and divide expressions with exponents.

So you can represent large amounts, such as distance in Ex. 89.

#### **KEY VOCABULARY**

• **exponent**, *p*. 19

• **power,** *p.* 19

#### Αστινιτγ

#### Use patterns to discover rules for multiplying powers.

	Expression	Expanded Expression	Number of Factors	Product as a Power	
	$2^2 \cdot 2^4$	(2 • 2) • (2 • 2 • 2 • 2)	6	2 <sup>6</sup>	
	$3^3 \cdot 3^1$	(3 • 3 • 3) • 3	?	3 <sup>?</sup>	
	$7^2 \cdot 7^3$	?	?	?	
<b>STEP 2 Explain</b> how the exponents in the first and last columns are related.					
Write th	e product as a	single power.			
1. $3^4 \cdot 3^3$		<b>2.</b> $6^5 \cdot 6^{11}$ <b>3.</b> 10		$10^7 \cdot 10^{13}$	

As you saw in the activity, you can expand expressions to find their product. The following equation suggests a rule for multiplying powers with the same base.

$$a^{4} \cdot a^{2} = \underbrace{(a \cdot a \cdot a \cdot a) \cdot (a \cdot a)}_{6 \text{ factors}} = a^{4+2} = a^{6}$$



	EXAMPLE 1	Using the Prod	luct of Powers Prope	rty
	Simplify $x^4 \cdot x^7$ .			
	$x^4 \cdot x^7 = x^4$	+ 7 <b>Product of po</b>	owers property	
	$= x^{1}$	Add exponer	nts.	
V	GUIDED PRACT	ICE for Example	21	
	Simplify the exp	pression. Write your	answer as a power.	
	<b>1.</b> $4^6 \cdot 4^4$	<b>2.</b> $9^8 \cdot 9$	<b>3.</b> $a^6 \cdot a^9$	<b>4.</b> $c \cdot c^{12} \cdot c^3$
(	EXAMPLE 2	Using the Prod	luct of Powers Prope	rty
	<b>33</b> Simplify $3^2x^2 \cdot 3^2$	$Bx^3$ .		
AVOID ERRORS	$3^2x^2 \cdot 3x^3 =$	$= (3^2 \cdot 3) \cdot (x^2 \cdot x^3)$	Use properties of multiplic	ation.
Remember that numbers raised to the first power	=	$= 3^{2+1} \cdot x^{2+3}$	Product of powers propert	у
are usually written without an exponent.	=	$= 3^3 x^5$	Add exponents.	
For example, $3 = 3^1$ .	=	$= 27x^5$	Evaluate the power.	
~	GUIDED PRACT	ICE for Example	2	
	Simplify the exp	pression.		
	<b>5.</b> $10^2 s^4 \cdot 10^4 s^2$	<b>6.</b> $6^3 t^5 \cdot 6^2 t^8$	<b>7.</b> $7x^2 \cdot 7x^4$	<b>8.</b> $5^2z \cdot 5z^7 \cdot z^2$
	Dividing Powers powers with the $\frac{a^5}{a^3} = \frac{a \cdot a}{a}$	The following equations same base when the factors $\frac{a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a \cdot a}{a \cdot a}$	ation suggests a rule for divergence of the exponents are integers. 2 factors $\frac{a^{1} \cdot a^{1}}{a^{2}} = a \cdot a = a^{5-3} = a^{2}$	iding 2
	🚬 KEY CON	СЕРТ	For Your No	otebook
	<b>Quotient</b> C	of Powers Proper	ty	
	Words	To divide two power subtract the expone exponent of the nur	rs with the same nonzero ba nt of the denominator from nerator.	se, the
	Algebra	$\frac{a^m}{a^n} = a^{m-n}$	<b>Numbers</b> $\frac{4^7}{4^4} = 4^{7-4}$	$4^{-4} = 4^3$

#### **EXAMPLE 3** Using the Quotient of Powers Property

3 Simplify the expression. Write your answer as a power.

**a.**  $\frac{x^{12}}{x^7} = x^{12-7}$ Quotient of powers property $= x^5$ Subtract exponents.**b.**  $\frac{9^7}{9^3} = 9^{7-3}$ Quotient of powers property $= 9^4$ Subtract exponents.

#### **EXAMPLE 4** Simplifying Fractions with Powers

#### Ѹ Simplify the expression.

**AVOID ERRORS a.**  $\frac{y^4 \cdot y}{y^3} = \frac{y^5}{y^3}$  $= y^{5-3}$ Simplify numerator using product of powers property. The bases of the powers must be the same to use **Quotient of powers property** the product or quotient property. In part (b) of  $= y^2$ Subtract exponents. Example 4, you cannot **b.**  $\frac{xy^4}{y^3} = xy^{4-3}$ simplify the numerator **Quotient of powers property** any further because the bases, x and y, are different. = xySubtract exponents.

**GUIDED PRACTICE**for Examples 3 and 4Simplify the expression. Write your answer as a power.9.  $\frac{a^6}{a^4}$ 10.  $\frac{10^9}{10^6}$ 11.  $\frac{q^3 \cdot q^5}{q^4}$ 12.  $\frac{a^2b^8}{b^2}$ HOMEWORK KEY**★** = STANDARDIZED TEST PRACTICE Exs. 31, 54, 88, 90, 91, and 108**●** = HINTS AND HOMEWORK HELP for Exs. 3, 17, 23, 33, 83 at classzone.comSKILL PRACTICEVOCABULARY Copy and complete the statement.1. Three is the  $\frac{2}{2}$  of the power  $3^4$ .2. Seven is the  $\frac{2}{2}$  of the power  $4^7$ .

### **SIMPLIFYING PRODUCTS** Simplify the product. Write your answer as a power.

SEE EXAMPLE 1	$3.4^2 \cdot 4^4$	<b>4.</b> $8 \cdot 8^3$	<b>5.</b> $a^5 \cdot a^7$	<b>6.</b> $b^9 \cdot b^9$
on p. 203 for Exs. 3–14	<b>7.</b> $u^7 \cdot u$	<b>8.</b> $v^2 \cdot v^{10}$	<b>9.</b> $b^9 \cdot b^6$	<b>10.</b> $m^{11} \cdot m^8$
-	<b>11.</b> $3^2 \cdot 3^5$	<b>12.</b> $7^2 \cdot 7^2$	<b>13.</b> $(-4)^2 \cdot (-4)^3$	14. $(-5)^4 \cdot (-5)$



**54. ★ OPEN-ENDED MATH** Write three variable expressions that simplify to  $x^4y^4$ . Write three variable expressions that simplify to  $\frac{2b^3}{5a}$ .

NUMBER SENSE Find the number that correctly completes the equation.

**55.**  $2^3 \cdot 2^? = 2^{11}$ **56.**  $5^4 \cdot ?^5 = 5^9$ **57.**  $12^3 \cdot ?^{13} = 12^{16}$ **58.**  $3^2 \cdot 3^? = 3^7$ **59.**  $\frac{8^7}{8^?} = 8^3$ **60.**  $\frac{7^4}{7^?} = 7$ **61.**  $\frac{13^?}{13^9} = 13^2$ **62.**  $9^3 \cdot 9^? = 9^{20}$ **63.**  $\frac{6^{14}}{6^?} = 6^6$ **64.**  $\frac{12^?}{12^5} = 12^4$ **65.**  $4^? \cdot 4^7 = 4^{19}$ **66.**  $\frac{11^3}{11^?} = 11^2$ 

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#### **PROBLEM SOLVING**

**METRIC UNITS** In Exercises 82–87, use the table. It shows the number of meters in some metric measures written as powers of ten. Write your answers as powers of 10.

- 82. How many decameters are in a yottameter?
- (83.) How many kilometers are in a petameter?
- 84. How many gigameters are in a zettameter?
- 85. How many terameters are in a yottameter?
- 86. How many megameters are in an exameter?
- 87. How many petameters are in a zettameter?
- **88.** ★ **MULTIPLE CHOICE** The product of 1 kilometer and which power of 10 yields 1 exameter? Use the table at the right.

(A) $10^6$	B	$10^{9}$
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- (**C**)  $10^{15}$  (**D**)  $10^{18}$
- **89. MULTI-STEP PROBLEM** The distance to the Andromeda Galaxy is 21 quintillion kilometers, which is 21 followed by 18 zeros. Refer to the table above to answer the questions.
  - a. How many exameters are in 1 quintillion kilometers?
  - **b.** Use your answer to part (a) to find the number of exameters in 21 quintillion kilometers.
  - c. How many gigameters are in 1 exameter?
  - **d.** Use your answers to parts (b) and (c) to find the number of gigameters in 21 quintillion kilometers.

Metric Units			
Unit	Meters		
yottameter	10 <sup>24</sup>		
zettameter	10 <sup>21</sup>		
exameter	10 <sup>18</sup>		
petameter	10 <sup>15</sup>		
terameter	10 <sup>12</sup>		
gigameter	10 <sup>9</sup>		
megameter	10 <sup>6</sup>		
kilometer	10 <sup>3</sup>		
decameter	10 <sup>1</sup>		



The historical standard platinum iridium meter bar

- **90. ★ SHORT RESPONSE** *Describe* two ways to evaluate the expression  $\frac{6y^{-1}}{3y^8}$  when y = 5. Evaluate the expression using both methods. Which method do you prefer? Why?
- **91.** ★ WRITING *Explain* why it is not necessary to multiply to evaluate powers of 10. Include an example with your answer.
- **92. CHECKING REASONABLENESS** It takes you  $5^2$  seconds to run *x* feet. If your speed remains constant, is it reasonable to say that it will take you  $5^3$  seconds to run 5*x* feet? *Justify* your answer.
- **93.** LAND AREA A square field has sides of length 3<sup>8</sup> millimeters. Find the area of the field. A larger field has side lengths 9 times those of the smaller field. How many times greater is the area of the large field? *Explain*.
- **94. COMPUTERS** A friend tells you that his family first bought a computer in 1979. It had 2<sup>3</sup> KB (kilobytes) of available memory.
  - **a.** Your family's first computer was purchased in 1987 and had 64 times the amount of memory of your friend's. How much memory did your computer have? Express your answer as a power of 2.
  - **b.** A typical computer purchased in 2005 had at least 2<sup>9</sup> MB (megabytes) of memory. A MB is equal to 2<sup>10</sup> KB. How many times the amount of the memory in your friend's 1979 computer is this? *Explain*.
- **95. CHALLENGE** A number *x* is *c* followed by *b* zeros. Another number *y*, which is smaller than *x*, is *d* followed by *b* zeros. How many times greater is *x* than *y*? *Explain* your reasoning.

#### **MIXED REVIEW**



# **4.7** Negative and Zero Exponents



You simplified expressions with positive exponents. You'll simplify expressions with negative exponents.

So you can describe very quick events, as the flash in Example 1.

Key Vocabulary • exponent, p. 19 • common factor, p. 181

Before

Now

Why?

You have seen two ways to evaluate expressions involving division of powers. Compare the results of the two methods when the denominator contains the greater power.

Divide out common factors.

Quotient of powers property

 $\frac{x^5}{x^7} = \frac{x^1 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^1}{x \cdot x \cdot x} = \frac{1}{x^2}$ 

 $\frac{x^5}{x^7} = x^{5-7} = x^{-2}$ 

So  $\frac{1}{x^2} = x^{-2}$ . This suggests the following definition for negative exponents.

11	KEY CO	NCEPT		For Your Notebook	
000	Negativ	e Exponents			
00	Words	For any integer <i>n</i> and any number $a \neq 0$ , $a^{-n}$ is equal to $\frac{1}{a^n}$ .			
0000	Algebra	$a^{-n} = \frac{1}{a^n}$	Numbers	$2^{-3} = \frac{1}{2^3}$	

#### **EXAMPLE 1** Using a Negative Exponent

**Strobes** The picture of the golf ball was taken using a strobe light. Each flash of the strobe light lasted about 1 microsecond. How can you write this time in seconds as a power of ten?

#### SOLUTION

The flash lasts 1 microsecond. Because the prefix *micro* means one millionth of a unit, you know that 1 microsecond is equal to  $\frac{1}{1.000.000}$  second.

1,000,00

 $\frac{1}{1,000,000} = \frac{1}{10^6}$ 

Write 1,000,000 as 10<sup>6</sup>.

 $= 10^{-6}$ 

**Definition of negative exponent** 

• Answer One flash of the strobe light lasts about  $10^{-6}$  second.



#### **EXAMPLE 3** Simplifying Variable Expressions

**33** Simplify. Write the expression using only positive exponents.

<b>a.</b> $-2n^0 = -2 \cdot n^0$	Zero exponent applies only to <i>n</i> .		
= -2 • <b>1</b>	Definition of zero exponent		
= -2	Multiply.		
<b>b.</b> $\frac{8x^{-3}}{x} = \frac{8 \cdot x^{-3}}{x^1}$	Exponent applies only to x.		
$= 8 \cdot x^{-3-1}$	Quotient of powers property		
$= 8 \cdot x^{-4}$	Simplify.		
$=\frac{8}{x^4}$	Definition of negative exponent		
Maimates Math at classzone.com			

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#### **GUIDED PRACTICE** for Examples 1, 2, and 3

**1. What If?** In Example 1, suppose the flash of a light lasted about one thousandth of a second. Write this time in seconds as a power of 10.

Evaluate the expression.

**2.**  $7^{-2}$  **3.**  $(-2)^{-5}$  **4.**  $6 \cdot 6^{-3}$  **5.**  $10^{-5} \cdot 10^{5}$ 

Simplify. Write the expression using only positive exponents.

**6.**  $-6m^{-1}$  **7.**  $b^2 \cdot b^{-2}$  **8.**  $\frac{5x^4}{x^{-7}}$  **9.**  $\frac{10a^{-3}}{a^4}$ 

AVOID ERRORS

In expressions such as  $-2n^0$  and  $4n^{-5}$ , the exponent is applied only to the variable, not to the coefficient.  $-2n^0 \neq -2^0 \cdot n^0$ 



#### **PROBLEM SOLVING**



#### MIXED REVIEW

Get-Ready	Write the number in expanded form. (p. 759)					
Prepare for Lesson 4.8 in	<b>46.</b> 32,501.5	47.805	55.93	<b>48.</b> 163.427		
Exs. 46–48	Evaluate the expression. (p. 19)					
	<b>49.</b> $(4 \times 3)^2 + 13$	<b>50.</b> 405	$5 \div (14 - 11)^4$	<b>51.</b> $96 \div 2^5 \times 6$		
	52. ★ MULTIPLE CH	<b>IOICE</b> Simplify the	expression $b^3 \cdot b^2$ .	( <b>p. 202</b> )		
	<b>A</b> 5b	( <b>B</b> ) $b^5$	<b>C</b> 6 <i>b</i>	( <b>D</b> ) $b^6$		
	EXTRA PRACTICE for	or Lesson 4.7, p. 804	n (	LINE QUIZ at classzone.com	211	